**CSCI 561 - Non-Programming Assignment 2**

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**8.9** This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

1. *Paris and Marseilles are both in France.*
2. In(Paris ∧ Marseilles, France ).

It is syntactically invalid since the term (Paris ∧ Marseilles) is not a geographical region.

1. In(Paris, France ) ∧ In(Marseilles, France ).

It is syntactically valid and correctly expresses the English sentence.

1. In(Paris, France ) ∨ In(Marseilles, France ).

It is syntactically valid but ∨ does not represent that Paris and Marseilles are both in France.

1. *There is a country that borders both Iraq and Pakistan.*
2. ∃ c Country(c) ∧ Border (c, Iraq) ∧ Border (c, Pakistan).

It is syntactically valid and correctly expresses the English sentence.

1. ∃ c Country(c) ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)].

It is syntactically valid but ⇒ does not express the meaning of the sentence.

1. [∃ c Country(c)] ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)].

It is syntactically invalid since c is used out of scope of ∃.

1. ∃ c Border (Country(c), Iraq ∧ Pakistan).

It is syntactically invalid since the term (Iraq ∧ Pakistan) is not a geographical region.

1. *All countries that border Ecuador are in South America.*
2. ∀c Country(c) ∧ Border (c,Ecuador ) ⇒ In(c, SouthAmerica).

It is syntactically valid and correctly expresses the English sentence.

1. ∀ c Country(c) ⇒ [Border (c,Ecuador ) ⇒ In(c, SouthAmerica)].

It is syntactically valid and also correctly expresses the English sentence.

1. ∀ c [Country(c) ⇒ Border (c,Ecuador )] ⇒ In(c, SouthAmerica).

It is syntactically valid but represents that if c is not a country, it is in South America.

1. ∀c Country(c) ∧ Border (c,Ecuador ) ∧ In(c, SouthAmerica).

It is syntactically valid but ∧ does not express the meaning of the sentence.

1. *No region in South America borders any region in Europe.*
2. ¬[∃ c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)].

It is syntactically valid and correctly expresses the English sentence.

1. ∀ c, d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒ ¬Borders(c, d)].

It is syntactically valid and also correctly expresses the English sentence.

1. ¬∀ c In(c, SouthAmerica) ⇒ ∃d In(d, Europe)∧ ¬Borders(c, d).

It is syntactically valid but does not represent the meaning of the sentence.

1. ∀ c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ¬Borders(c, d).

It is syntactically valid and also correctly expresses the English sentence.

1. *No two adjacent countries have the same map color.*
2. ∀ x, y ¬Country(x) ∨ ¬Country(y) ∨ ¬Borders(x, y) ∨

¬ (MapColor (x) = MapColor (y)).

It is syntactically valid and correctly expresses the English sentence.

1. ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y) ∧ ¬ (x = y)) ⇒

¬ (MapColor (x) = MapColor (y)).

It is syntactically valid and also correctly expresses the English sentence.

1. ∀ x, y Country(x) ∧ Country(y) ∧ Borders(x, y) ∧

¬ (MapColor (x) = MapColor (y)).

It is syntactically valid but ∧ does not represent the meaning of the sentence.

1. ∀ x, y (Country(x) ∧ Country(y) ∧ Borders (x, y)) ⇒ MapColor (x ≠ y).

It is syntactically invalid since (x ≠ y) is not a geographical region.

**8.20** Arithmetic assertions can be written in first-order logic with the predicate symbol <,

the function symbols + and ×, and the constant symbols 0 and 1. Additional predicates can

also be defined with biconditionals.

1. *Represent the property “x is an even number.”*

∀ n Even(n) ⇔ ∃ a (n = a + a).

1. *Represent the property “x is prime.”*

∀ n Prime(n) ⇔ ∀ a, b (n = a × b) ⇒ (a = 1) ∨ (b = 1).

1. *Goldbach’s conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.*

∀ n Even(n) ⇒∃ a, b Prime(a) ∧ Prime(b) ∧ (n = a + b).

**9.9** Suppose you are given the following axioms:

1. 0 ≤ 3.
2. 7 ≤ 9.
3. ∀x x≤ x.
4. ∀x x≤ x + 0.
5. ∀x x+ 0 ≤ x.
6. ∀ x, y x + y ≤ y + x.
7. ∀ w, x, y, z w ≤ y ∧ x ≤ z ⇒ w + x ≤ y + z.
8. ∀ x, y, z x ≤ y ∧ y ≤ z ⇒ x ≤ z
9. *Give a backward-chaining proof of the sentence 7 ≤ 3 + 9. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.*

**Backward Chaining**

*Goal 1-> 7 ≤ 3 + 9*

Resolving A with 8 substituting x/7, z/3 + 9, y/y1, we get 2 sub-goals

*Goal 1.1 -> 7 ≤ y1*

*Goal 1.2 -> y1 ≤ 3 + 9*

Resolving Goal 1.1 with 4 substituting x/7, we get the result,

7 ≤ 7 + 0 which is true.

∴ Goal 1.1 is achieved.

Also, y1 = 7 + 0.

*∴ Goal 1.2 -> 7 + 0 ≤ 3 + 9*

Resolving Goal 1.2 with 8 substituting x/(7 + 0), y/y2 and z/(3 + 9), we get 2 sub-goals

*Goal 1.2.1 -> 7 + 0 ≤ y2*

*Goal 1.2.2 -> y2 ≤ 3 + 9*

Resolving Goal 1.2.1 with 6 substituting x/7, y/0, we get the result.

7 + 0 ≤ 0 + 7 which is true.

∴ Goal 1.2.1 is achieved.

Also, y2 = 0 + 7.

*∴ Goal 1.2.2 -> 0 + 7 ≤ 3 + 9*

Resolving Goal 1.2.2 with 7 substituting w/0, x/7, y/3 and z/9, we get 2 sub-goals

*Goal 1.2.2.1 -> 0 ≤ 3*

*Goal 1.2.2.2 -> 7 ≤ 9*

Resolving Goals 1.2.2.1 and 1.2.2.2 with 1 and 2, we find that both are true.

∴ Goal 1.2.2 is achieved.

∴ Both Goals 1.2.1 and 1.2.2 are achieved.

∴ Goal 1 -> 7 ≤ 3 + 9 must be true.

1. *Give a forward-chaining proof of the sentence 7 ≤ 3 + 9. Again, show only the steps that lead to success.*

**Forward Chaining**

Resolving 1 and 2 with 7 substituting w/0, x/7, y/3, z/9, we get,

1. 0 + 7 ≤ 3 + 9

Resolving A with 6 substituting x/7 and y/0

1. 7 + 0 ≤ 0 + 7

Resolving B and A with 8 substituting x/(7 + 0), y/(0 + 7) and z/(3 + 9)

1. 7 + 0 ≤ 3 + 9

Substituting x/(7) in 4,

1. 7 ≤ 7 + 0

Resolving C and D with 8 substituting x/(7), y/(7 + 0) and z/(3 + 9), we get,

7 ≤ 3 + 9

**10.4** The original STRIPS planner was designed to control Shakey the robot. Figure 10.14

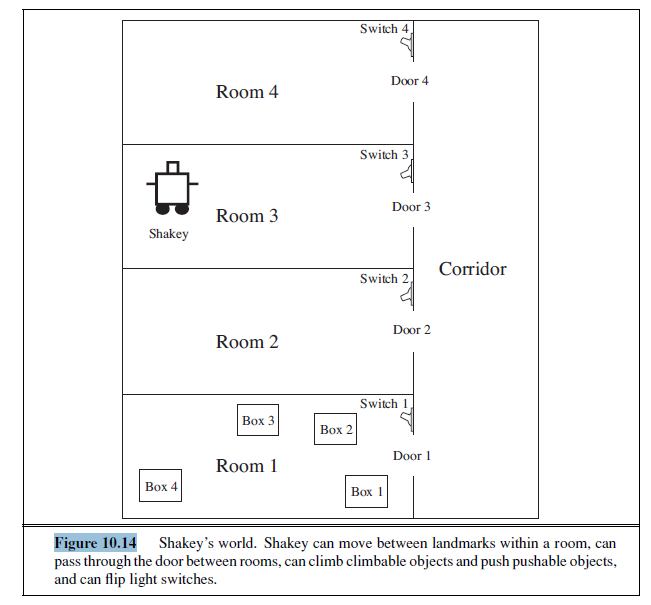
shows a version of Shakey’s world consisting of four rooms lined up along a corridor, where

each room has a door and a light switch. The actions in Shakey’s world include moving from

place to place, pushing movable objects (such as boxes), climbing onto and down from rigid objects (such as boxes), and turning light switches on and off. The robot itself could not climb

on a box or toggle a switch, but the planner was capable of finding and printing out plans that

were beyond the robot’s abilities. Shakey’s six actions are the following:



**PDDL sentences for Shakey’s six actions:**

* Go(x, y, r), which requires that Shakey be At x and that x and y are locations In the same room r. By convention a door between two rooms is in both of them.

*Action* (Go(x, y, r),

*Precondition*: At(Shakey, x) ∧ In(x, r) ∧ In(y, r),

*Effect*: At(Shakey, y) ∧ ¬(At(Shakey, x))

* Push a box b from location x to location y within the same room: Push(b, x, y, r). You will need the predicate Box and constants for the boxes.

*Action* (Push(b, x, y, r),

*Precondition*: At(Shakey, x) ∧ IsPushable(b) ∧ In(x, r) ∧ In(y, r),

*Effect*: At(b, y) ∧ At(Shakey, y) ∧ ¬At(b, x) ∧ ¬At(Shakey, x))

* Climb onto a box from position x: ClimbUp(x, b); climb down from a box to position x: ClimbDown(b, x). We will need the predicate On and the constant Floor .

*Action* (ClimbUp(x, b),

*Precondition*: At(Shakey, x) ∧ At(b, x) ∧ IsClimbable(b),

*Effect*: On(Shakey, b) ∧ ¬On(Shakey, Floor))

*Action* (ClimbDown(b, x),

*Precondition*: At(Shakey, x) ∧ At(b, x) ∧ On(Shakey, b),

*Effect*: On(Shakey, Floor) ∧ ¬On(Shakey, b))

* Turn a light switch on or off: TurnOn(s, b); TurnOff (s, b). To turn a light on or off, Shakey must be on top of a box at the light switch’s location.

*Action* (TurnOn(s, b),

*Precondition*: At(Shakey, x) ∧ At(b, x) ∧ At(l, x) ∧ On(Shakey, b),

*Effect*: IsTurnedOn(l))

*Action* (TurnOff(s, b),

*Precondition*: At(Shakey, x) ∧ At(b, x) ∧ At(l, x) ∧ On(Shakey, b),

*Effect*: ¬IsTurnedOn(l))

**The initial state from Figure 10.14:**

*//Room 1*

In(Switch1, Room1) ∧ In(Door1, Room1) ∧

In(Box1, Room1) ∧ In(Box2, Room1) ∧ In(Box3, Room1) ∧ In(Box4, Room1) ∧

IsClimbable(Box1) ∧ IsClimbable(Box2) ∧ IsClimbable(Box3) ∧ IsClimbable(Box4) ∧

IsPushable(Box1) ∧ IsPushable(Box2) ∧ IsPushable(Box3) ∧ IsPushable(Box4) ∧

At(Box1, x1) ∧ At(Box2, x2) ∧ At(Box3, x3) ∧ At(Box4, x4) ∧

IsTurnedOn(Switch1) ∧

*//Room 2*

In(Switch2, Room2) ∧ In(Door2, Room2) ∧

¬IsTurnedOn(Switch2)) ∧

*//Room 3*

In(Switch3, Room3) ∧ In(Door3, Room3) ∧

In(Shakey, Room3) ∧ At(Shakey, xS) ∧

¬IsTurnedOn(Switch3)) ∧

*//Room 4*

In(Switch4, Room4) ∧ In(Door4, Room4) ∧

IsTurnedOn(Switch4)

*//Corridor*

In(Door1, Corridor) ∧ In(Door2, Corridor) ∧ In(Door3, Corridor) ∧ In(Door4, Corridor)

**A plan for Shakey to get Box 2 into Room2:**

1. Go(xS, Door3, Room3)
2. Go(Door3, Door1, Corridor)
3. Go(Door1, x2, Room2)
4. Push(Box2, x2, Door1, Room1)
5. Push(Box2, Door1, Door2, Corridor)
6. Push(Box2, Door2, Switch2, Room2)

**13.8** Given the full joint distribution shown in Figure 13.3, calculate the following:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | toothache | | ¬toothache | |
|  | catch | ¬catch | catch | ¬catch |
| cavity | 0.108 | 0.012 | 0.072 | 0.008 |
| ¬cavity | 0.016 | 0.064 | 0.144 | 0.576 |
| **Figure 13.3 A full joint distribution for the Toothache, Cavity, Catch world.** | | | | |

1. ***P****(toothache).*

**P**(toothache) = 0.108 + 0.016 + 0.012 + 0.064= 0.2

**P**(toothache) = 0.2

1. ***P****(Cavity).*

There are 2 possible values for cavity - True, False.

P(Cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2

P(¬Cavity) = 0.016 + 0.064 + 0.144 + 0.576 = 0.8

**P**(Cavity) = <0.2, 0.8>

1. ***P****(Toothache | cavity).*

There are 2 possible values for Toothache | cavity - True, False.

P(toothache | cavity) = (0.108 + 0.012)/0.2 = 0.6

P(¬toothache | cavity) = (0.072 + 0.008)/0.2 = 0.4

**P**(Cavity) = <0.6, 0.4>

1. *P(Cavity | toothache ∨ catch).*

P(toothache ∨ catch) = 0.108 + 0.016 + 0.012 + 0.064 + 0.072 + 0.144 = 0.416

There are 2 possible values for Cavity | toothache ∨ catch - True, False.

P(cavity | toothache ∨ catch) = (0.108 + 0.012 + 0.072)/0.416 = 0.4615

P(¬cavity | toothache ∨ catch) = (0.016 + 0.064 + 0.144)/0.416 = 0.5384

**P**(Cavity | toothache ∨ catch) = <0.4615, 0.5384>

**13.10** Deciding to put probability theory to good use, we encounter a slot machine with

three independent wheels, each producing one of the four symbols BAR, BELL, LEMON, or

CHERRY with equal probability. The slot machine has the following payout scheme for a bet

of 1 coin (where “?” denotes that we don’t care what comes up for that wheel):

BAR/BAR/BAR pays 20 coins

BELL/BELL/BELL pays 15 coins

LEMON/LEMON/LEMON pays 5 coins

CHERRY/CHERRY/CHERRY pays 3 coins

CHERRY/CHERRY/? pays 2 coins

CHERRY/?/? pays 1 coin

1. *Compute the expected “payback” percentage of the machine. In other words, for each coin played, what is the expected coin return?*

There are 4 symbols and each is equally likely on each wheel. Consequently, we have a total of

4 × 4 × 4 = 64 possibilities

P(BAR/BAR/BAR) = 1/64

P(BELL/BELL/BELL) = 1/64

P(LEMON/LEMON/LEMON) = 1/64

P(CHERRY/CHERRY/CHERRY) = 1/64

Now, for P(CHERRY/CHERRY/?) we need to only consider the cases where ? ≠ CHERRY

∴ P(CHERRY/CHERRY/?) = (1/4) × (1/4) - 1/64

∴ P(CHERRY/CHERRY/?) = 3/64

Similarly, for P(CHERRY/?/?) we need to only consider the cases where both ? ≠ CHERRY

∴ P(CHERRY/?/?) = (1/4) - 3/64 - 1/64

∴ P(CHERRY/?/?) = 12/64

∴ Expectation = (1/64 × 20) + (1/64 × 15) + (1/64 × 5) + (1/64 × 3) + (3/64 × 2) + (12/64 × 1)

∴ Expectation = 61/64

1. *Compute the probability that playing the slot machine once will result in a win.*

P(WIN) = P(BAR/BAR/BAR) + P(BELL/BELL/BELL) + P(LEMON/LEMON/LEMON) + P(CHERRY/CHERRY/CHERRY) + P(CHERRY/CHERRY/?) + P(CHERRY/?/?)

∴ P(WIN) = (1/64) + (1/64) + (1/64) + (1/64) + (3/64) + (12/64)

∴ P(WIN) = 19/64

**13.15** After your yearly checkup, the doctor has bad news and good news. The bad news

is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the

probability of testing positive when you do have the disease is 0.99, as is the probability of

testing negative when you don’t have the disease). The good news is that this is a rare disease,

striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare?

What are the chances that you actually have the disease?

Let TEST+ denote that the test was positive for the disease and TEST- denote that the test was negative for the same disease.

P(TEST+ | DISEASE) = 0.99

∴ P(TEST+ | ¬DISEASE) = 0.01

P(TEST- | ¬DISEASE) = 0.99

∴ P(TEST- | DISEASE) = 0.01

P(DISEASE) = 0.0001

∴ P(¬DISEASE) = 0.9999

According to Bayes Rule:-

P(DISEASE | TEST+) = ( P(TEST+ | DISEASE) × P(DISEASE) ) / P(TEST+)

Now, P(TEST+) = P(TEST+ | DISEASE) × P(DISEASE) + P(TEST+ | ¬DISEASE) × P(¬DISEASE)

∴ P(TEST+) = 0.99 × 0.0001 + 0.01 × 0.9999

∴ P(TEST+) = 0.010098

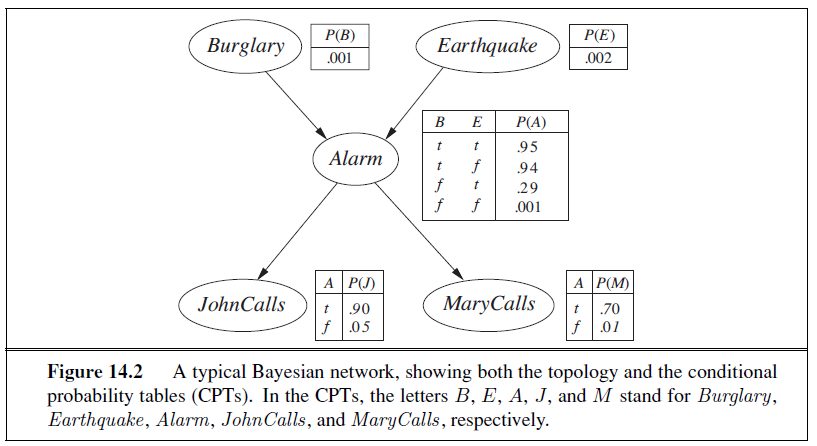
∴ P(DISEASE | TEST+) = (0.99 × 0.0001) / (0.010098)

∴ P(DISEASE | TEST+) = 0.009804.

Thus, even though the probability of testing positive for the disease given that one has the disease is high, due to the rarity of the disease, the probability of having the disease given that the test was positive is very low (less than 1%). Hence, it is good news that the disease is rare.

The probability that we actually have the disease given that the test turned out positive is 0.009804.

**14.4** Consider the Bayesian network in Figure 14.2.



1. *If no evidence is observed, are Burglary and Earthquake independent? Prove this from the numerical semantics and from the topological semantics.*

**Numerical Semantics:**

In a Bayesian network, we have,

P(BURGLARY, EARTHQUAKE) = P(BURGLARY | PARENT(BURGLARY)) ×

P(EARTHQUAKE, | PARENT(EARTHQUAKE))

Since both BURGLARY and EARTHQUAKE have no parents,

∴ P(BURGLARY, EARTHQUAKE) = P(BURGLARY) × P(EARTHQUAKE)

∴ BURGLARY and EARTHQUAKE are independent based on numerical semantics.

**Topological Semantics:**

In a Bayesian network, each variable is conditionally independent of its non-descendants, given its parents. From the given figure, either one of BURGLARY and EARTHQUAKE is not a descendants of the other.

∴ BURGLARY and EARTHQUAKE are independent based on topological semantics.

1. *If we observe Alarm =true, are Burglary and Earthquake independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence.*

In order to check if they are independent given that ALARM = TRUE, we need to prove that

P(B, E | A) = P(B | A) × P(E | A)

1. P(A) = P(A | B, E) P(B) P(E) +

P(A | B, ¬ E) P(B) P(¬E) +

P(A | ¬ B, E) P(¬B) P(E) +

P(A | ¬ B, ¬ E) P(¬B) P(¬E)

∴P(A) = (0.95 × 0.001 × 0.002) +

(0.94 × 0.001 × 0.998) +

(0.29 × 0.999 × 0.002) +

(0.001 × 0.999 × 0.998)

∴P(A) = 0.002516

1. P(B | A) = (P(A | B) × P(B)) / P(A)

∴ P(B | A) = (P(A | B, E) P(B) P(E) + P(A | B, ¬E) P(B)P(¬E)) / P(A)

∴ P(B | A) = (0.95 × 0.001 × 0.002 + 0.94 × 0.001 × 0.998) / 0.002516

∴ P(B | A) = 0.37451

1. P(E | A) = P(A, E) × P(A)

∴ P(E | A) = (P(A, E | B) P(B) + P(A, E | ¬B) P(¬B)) /P(A)

∴ P(E | A) = (P(A | B, E) P(B) P(E) + P(A | ¬B, E) P(¬B)) /P(A)

∴ P(E | A) = (0.95 × 0.001 × 0.002 + 0.29 × 0.999 × 0.002) / 0.002516

∴ P(E | A) = 0.2316

1. ∴ P(B | A) × P(E | A) = 0.37451 × 0.2316 = 0.0867
2. P(B, E | A) = (P(A | B, E) × P(B, E))/P(A)

∴ P(B, E | A) = (P(A | B, E) × P(B) × P(E))/P(A)

∴ P(B, E | A) = (0.95 × 0.001× 0.002)/0.002516

∴ P(B, E | A) = 0.000757

∴ From 4 and 5,

P(B, E | A) ≠ P(B | A) × P(E | A)

∴ If we observe ALARM = TRUE, BURGLARY and EARTHQUAKE are not independent.

**14.6** Let Hx be a random variable denoting the handedness of an individual x, with possible

values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple

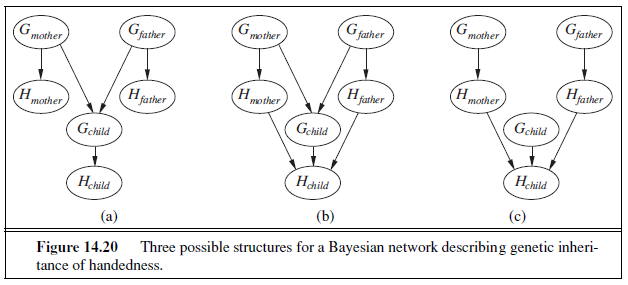
mechanism; that is, perhaps there is a gene Gx, also with values l or r, and perhaps actual

handedness turns out mostly the same (with some probability s) as the gene an individual

possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either

of an individual’s parents, with a small nonzero probability m of a random mutation flipping

the handedness.



1. *Which of the three networks in Figure 14.20 claim that*

*P(Gfather,Gmother,Gchild) = P(Gfather)P(Gmother)P(Gchild)?*

The network (c) denotes this particular equation since in c), Gfather,Gmother and Gchild are each independent of each other.

1. *Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?*

In both networks (a) and (b),

1. Gchild is dependent on Gfather and Gmother.
2. Hchild is dependent on Gchild.

This is consistent with the hypothesis about the inheritance of handedness.

In figure (c),

Gchild is independent of both Gfather and Gmother which goes against the hypothesis.

1. *Which of the three networks is the best description of the hypothesis?*

The network (a) is the best description of the hypothesis since the network (b) also denotes that Hchild is dependent on Hfather and Hmother.

This is an extra dependency not specified in the hypothesis.

1. *Write down the CPT for the Gchild node in network (a), in terms of s and m.*

m is the probability of flipping of handedness.

In the case where both parents have the same gene, there is a probability of m for mutation of the child's gene and a probability of 1 - m for the child to have the same gene.

On the other hand, when the parents have different genes, the child's gene can follow either one with a probability of 0.5 as shown.

|  |  |  |  |
| --- | --- | --- | --- |
| **Gm** | **Gf** | **P(Gchild = l | Gmother = Gm** ∧**Gfather = Gm)** | **P(Gchild = r | Gmother = Gm** ∧**Gfather = Gm)** |
| l | l | 1 - m | m |
| l | r | 0.5(1 - m) + 0.5m = 0.5 | 0.5m + 0.5(1 - m) = 0.5 |
| r | l | 0.5m + 0.5(1 - m) = 0.5 | 0.5(1 - m) + 0.5m = 0.5 |
| r | r | M | 1 - m |

1. *Suppose that P(Gfather = l) = P(Gmother = l) = q. In network (a), derive an expression for*

*P(Gchild = l) in terms of m and q only, by conditioning on its parent nodes.*

Since Gfather and Gmother are independent,

P(Gchild = l) = P(Gchild = l | Gfather = l ∧ Gmother = l) × P(Gfather = l) × P(Gmother = l) +

P(Gchild = l | Gfather = l ∧ Gmother = r) × P(Gfather = l) × P(Gmother = r) +

P(Gchild = l | Gfather = r ∧ Gmother = l) × P(Gfather = r) × P(Gmother = l) +

P(Gchild = l | Gfather = r ∧ Gmother = r) × P(Gfather = r) × P(Gmother = r)

∴ P(Gchild = l) = (1 - m) q2 +

0.5q(1 - q) +

0.5(1 - q)q +

m(1 - q)2 +

∴ P(Gchild = l) = q + m - 2mq

1. *Under conditions of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this to calculate the value of q, and, given what you know about handedness in humans, explain why the hypothesis described at the beginning of this question must be wrong.*

Under conditions of genetic equilibrium,

P(Gchild = l) = P(Gmother = l) = P(Gfather = l)

∴ q + m - 2mq = q

∴ m(1 - 2q) = 0

Since m ≠ 0, q = 0.5

This implies that P(Gchild = l) = P(Gchild = r) = 0.5.

However, there are extremely few left-handed people compared to right-handed people in the world today. Consequently, the hypothesis provided must be wrong.

**14.11** In your local nuclear power station, there is an alarm that senses when a temperature

gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider

the Boolean variables A(alarm sounds), FA(alarm is faulty), and FG(gauge is faulty) and

the multivalued nodes G(gauge reading) and T(actual core temperature).

1. *Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.*
2. *Is your network a polytree? Why or why not?*

The network is NOT a polytree since there are two paths from T to G:-

T -> G

T -> FG -> G

1. *Suppose there are just two possible actual and measured temperatures, normal and high;*

*the probability that the gauge gives the correct temperature is x when it is working, but*

*y when it is faulty. Give the conditional probability table associated with G.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **T = normal** | | **T = high** | |
| **FG** | ¬**FG** | **FG** | ¬**FG** |
| **G = normal** | y | x | 1 - y | 1 - x |
| **G = high** | 1 - y | 1 - x | y | x |

1. *Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **G = normal** | | **G = high** | |
| **FA** | ¬**FA** | **FA** | ¬**FA** |
| **A** | false | false | false | true |
| ¬**A** | true | true | true | false |

1. *Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.*

Since the alarm and gauge are working, we have FA = ¬fA and FG = ¬fG.

Also, since the alarm sounds, we have A = a.

We need to find the probability that the temperature is high (T = t) i.e.

P(T = t | A = a, FA = ¬fA, FG = ¬fG) = P(t | a, ¬fA, ¬fG)

Since the alarm sounds, it is not faulty.

∴ From the table in d, we get, G = high. Let us denote this as g.

∴ P(t | a, ¬fA, ¬fG) now becomes P(t | a, ¬fA, g, ¬fG)

Applying Bayes Rule,

P(t | a, ¬fA, g, ¬fG) = P(t , a, ¬fA, g, ¬fG) / P(a, ¬fA, g, ¬fG)

Since a and fA are independent,

∴ P(t | a, ¬fA, g, ¬fG) = P(a | ¬fA, g) P(¬fA) P(g | ¬fG, t) P(¬fG | t) P(t)

P(a | ¬fA, g) P(¬fA) P(g | ¬fG) P(¬fG)

∴ P(t | a, ¬fA, g, ¬fG) = P(g | ¬fG, t) P(¬fG | t) P(t)

P(g | ¬fG) P(¬fG)

∴ P(t | a, ¬fA, g, ¬fG) = P(g | ¬fG, t) P(¬fG | t) P(t)

P(g, ¬fG)

∴ P(t | a, ¬fA, g, ¬fG) = P(g | ¬fG, t) P(¬fG | t) P(t)

P(t, g, ¬fG) + P(¬t, g, ¬fG)

∴ P(t | a, ¬fA, g, ¬fG) = P(g | ¬fG, t) P(¬fG | t) P(t)

P(g | ¬fG, t) P(¬fG | t) P(t) + P(g | ¬fG, ¬t) P(¬fG, ¬t) P(¬t)

Assuming P(t) = m, P(fG | t) = n and P(fG | ¬t) = p

∴ P(¬fG | t) = 1 - n

P(¬fG | ¬t) = 1 - p

P(¬t) = 1 - m

Also, from the CPT in part c,

P(g | ¬fG, t) = 1 - x

P(g | ¬fG, ¬t) = x

∴ P(t | a, ¬fA, g, ¬fG) = (1 - x) (1 - n) m

(1 - x) (1 - n) m + x(1 - p) (1- m)